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$1/N_c$ Rotational Corrections to g_A and Isovector Magnetic Moment of the Nucleon

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(February 1, 2008)

Abstract

The $1/N_c$ rotational corrections to the axial vector constant and the isovector magnetic moment of the nucleon are studied in the Nambu – Jona-Lasinio model. We follow a semiclassical quantization procedure in terms of path integrals in which we can include perturbatively corrections in powers of angular velocity $\Omega \sim \frac{1}{N_c}$. We find non-zero $1/N_c$ order corrections from both the valence and the Dirac sea quarks. These corrections are large enough to resolve the long-standing problem of a strong underestimation of both g_A and μ^{IV} in the leading order. The axial constant g_A is well reproduced, whereas the isovector magnetic moment μ^{IV} is still underestimated by 25 %.

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Among the effective low-energy models of baryons the chiral soliton model, based on a semibosonized version [1] of the Nambu – Jona-Lasinio lagrangean [2], seems to play an essential role. In fact, an equivalent effective chiral quark-meson lagrangean can be derived [3] from the instanton model of the QCD vacuum. This chiral soliton model, which is frequently referred simply as NJL model, incorporates the general accepted phenomenological picture of the baryon as a bound state of N_c valence quarks coupled to the polarized Dirac sea ($\overline{q}q$ pairs). Operationally in the model the nucleon problem is solved in two steps [4]. In the first step, motivated by the large N_c (number of colors) limit, a static localized solution (soliton) of a hedgehog structure is found. This hedgehog solution does not preserve the spin and isospin. Making use of the rotational zero modes in a semiclassical quantization scheme one can assign proper spin and isospin quantum numbers to the soliton. The model is rather successful in describing both the static nucleon properties [4–8] and the nucleon form factors [9,10]. The only exceptions are the axial coupling constant g_A and the isovector magnetic moment μ^{IV}, which are strongly underestimated ($g_A \approx 0.8$ and $\mu^{IV} \approx 2.4$(n.m.)) in the leading order, compared to the experimental values of 1.25 and 4.71 (n.m.). This is not a problem just of the NJL model. In fact, most of the chiral models, if they assume physical values of the pion decay constant f_π and the pion mass m_π, yield rather strong deviations from the experimental values for these quantities. In the case of the NJL model the simplest solution to this problem is to take into account the rotational corrections coming from the next to leading order terms.

In a very recent paper Wakamatsu and Watabe [11] estimated in the NJL model $1/N_c$ rotational corrections to g_A thus providing a step in the right direction. They found a considerable non-zero valence contribution ($\approx$ 0.4). In this work they made the important observation that in the case of g_A, after the canonical quantization, the collective operators do not commute. However, in their scheme the non-zero result is

due to a particular order of the collective operators being not fully justified by path integrals or many-body techniques. As a consequence the valence contribution includes transitions between occupied levels, which violate the Pauli principle (even though, numerically, this Pauli violating contribution turns out to be only a tiny fraction of the valence contribution), and the Dirac sea $1/N_c$ contribution vanishes exactly. The latter is particularly puzzling since apart from the regularization there is no any principle difference between the valence and the sea quarks in the NJL model. Based on the formulae of Wakamatsu and Watabe [11] Alkofer and Weigel [12] studied the axial coupling constant in the context of the PCAC. From their values for g_A one can estimate how far the PCAC holds up in the linear order in Ω and in particular, a violation of less than 2 % for a reasonable value of the constituent mass $M = 400$ MeV can be guessed.

Obviously a satisfying theoretical and numerical treatment of higher order rotational corrections in the semiclassical quantization scheme is still missing. It is therefore the objective of the present work to evaluate the $1/N_c$ rotational corrections to both the axial coupling constant and the isovector magnetic moment in the NJL model. To that end we will follow the theoretical scheme of Diakonov et al. [4] elaborated in terms of path integrals.

We start with the definitions of axial and isovector magnetic currents of a fermion field $\Psi(x)$, $A_k^a(x) = \Psi^+(x)\gamma_0\gamma_k\gamma_5\frac{\tau^a}{2}\Psi(x)$ and $J_k^a(x) = \Psi^+(x)\gamma_0\gamma_k\frac{\tau^a}{2}\Psi(x)$, respectively. Here k means space components and a stands for the isospin index. We express the nucleon matrix element of a current operator $\hat{A}_k^a$ as a path integral including quark Ψ, Ψ^+ and meson fields U in Minkowski space:

$$
\begin{aligned}
\langle N(\mathbf{p\prime})\,|\Psi^+(0)\hat{O}_k^a\Psi(0)|\,N(\mathbf{p})\rangle &\underset{T\to -i\infty}{=} \tfrac{1}{Z}\int d^3x d^3y e^{-i\mathbf{p}'\cdot\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{y}} \\
\times \int \mathcal{D}U \int \mathcal{D}\Psi\mathcal{D}\Psi^+ J_N(T/2,\mathbf{x}) J_N^+(-T/2,\mathbf{y})\Psi^+(0)\hat{O}_k^a\Psi(0) e^{i\int d^4z \Psi^+ D(U)\Psi} & \qquad (1)
\end{aligned}
$$

The equality (1) should be understood as a limit at large euclidean time separation. Here

Z is the normalization factor which is related to the same path integral but without the current operator $\Psi^+\hat{O}_k^a\Psi$, and $\hat{O}_k^a$ stands for the matrix part $\gamma_0\gamma_k(\gamma_5)\frac{\tau^a}{2}$ of the current. The Dirac operator $D(U) = i\partial_t - h(U)$ includes the single-particle hamiltonian $h(U) = \frac{\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}}{i} + M\beta U^{\gamma_5} + m_0\beta$ with meson fields $U^{\gamma_5} = e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}}$. Here $\boldsymbol{\alpha}$ and β are the Dirac matrices and m_0 being the current quark mass. The composite operator:

$$J_N(x) = \frac{1}{N_c!}\varepsilon^{\beta_1\cdots\beta_{N_c}}\Gamma^{\{f_1\cdots f_{N_c}\}}_{JJ_3,TT_3}\Psi_{\beta_1 f_1}(x)\cdots\Psi_{\beta_{N_c}f_{N_c}}(x), \tag{2}$$

carries the quantum numbers JJ_3, TT_3 (spin, isospin) of the nucleon, where β_i is the color index, and $\Gamma^{f_1\cdots f_{N_c}}_{JJ_3,TT_3}$ is a symmetric matrix in flavor and spin indices f_i.

In eq.(1) we can integrate the quarks out:

$$\begin{aligned}
\langle N(\mathbf{p}')\,|\Psi^+(0)\hat{O}_k^a\Psi(0)|\,N(\mathbf{p})\rangle &= \tfrac{1}{Z}\Gamma^{\{g\}}_{JJ_3,TT_3}\Gamma^{\{f\}}_{JJ_3,TT_3}N_c\int d^3x d^3y e^{-i\mathbf{p}'\cdot\mathbf{x}}e^{i\mathbf{p}\cdot\mathbf{y}}\int\mathcal{D}U \\
\times\{\langle T/2,\mathbf{x}\,|\tfrac{i}{D}|\,0,0\rangle_{f_1 f'}(\hat{O}_k^a)_{f'g'}&\langle 0,0\,|\tfrac{i}{D}|\,-T/2,\mathbf{y}\rangle_{g'g_1} - \mathrm{Sp}(\langle 0,0\,|\tfrac{i}{D}|\,0,0\rangle\hat{O}_k^a) \\
\times\langle T/2,\mathbf{x}\,|\tfrac{i}{D}|\,-T/2,\mathbf{y}\rangle_{f_1g_1}\}&\prod_{i=2}^{N_c}\langle T/2,\mathbf{x}\,|\tfrac{i}{D}|\,-T/2,\mathbf{y}\rangle_{f_ig_i}e^{\mathrm{tr}\log D(U)}.
\end{aligned} \tag{3}$$

In a natural way the result is split in a valence – the first term, and a Dirac sea contribution – the second one (see the diagrams on the l.h.s. of Fig.1 a) and b)).

In order to integrate over the meson fields U we start from a stationary meson configuration of hedgehog structure $\bar{U}(x) = e^{i\boldsymbol{\tau}\cdot\hat{\mathbf{x}}\pi(x)}$ which minimizes the effective action. Then the integration over the meson fields U in the path integral can be done in a saddle point approximation, which is motivated by the large N_c limit. In the next step we should allow the system to fluctuate around the static hedgehog solution $\bar{U}(x)$ making use of the rotational zero modes. Since the fluctuations which correspond to the zero modes are not small they have to be treated "exactly" in the meaning of path integral. Operationally it can be done introducing a rotating meson fields of the form $U(\mathbf{x},t) = R(t)\bar{U}(\mathbf{x})R^+(t)$, where $R(t)$ is a time-dependent rotation SU(2) matrix in the isospin space. It is easy to see that for such an ansatz one can transform the

effective action $\quad \mathrm{tr}\log D(U) = \mathrm{tr}\log(D(\bar{U}) - \Omega) \quad$ as well as the quark propagator in the background meson fields U

$$\langle T/2, \mathbf{x}\,|\frac{i}{D(U)}|-T/2, \mathbf{y}\rangle = R(T/2)\langle T/2, \mathbf{x}\,|\frac{i}{D(\bar{U}) - \Omega}|-T/2, \mathbf{y}\rangle R^{+}(-T/2), \qquad (4)$$

where $\Omega = -iR^{+}(t)\dot{R}(t) = \frac{1}{2}\Omega_a\tau_a$ is the angular velocity matrix. Since $\Omega \sim \frac{1}{N_c}$ (as can be seen below) one can consider Ω as a perturbation and evaluate any observable as a perturbation series in Ω which is actually an expansion in $\frac{1}{N_c}$.

In this scheme the matrix element (eq.1) of the current can be written as

$$\begin{aligned}
&\langle N(\mathbf{p'})\,|\Psi^{+}(0)\hat{O}^a_k\Psi(0)|\,N(\mathbf{p})\rangle = \tfrac{1}{Z}N_c\int d^3x d^3y d^3z e^{-i\mathbf{p}'\cdot\mathbf{x}}e^{i\mathbf{p}\cdot\mathbf{y}}e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{z}}\int\mathcal{D}R\\
&\times D^{J}_{-T_3J_3}{}^{*}(R(T/2))\{\langle T/2, \mathbf{x}\,|\tfrac{i}{D-\Omega}|\,0, \mathbf{z}\rangle R^{+}(0)\hat{O}^a_k R(0)\langle 0, \mathbf{z}\,|\tfrac{i}{D-\Omega}|-T/2, \mathbf{y}\rangle\\
&\quad - \mathrm{Sp}(\langle 0, \mathbf{z}\,|\tfrac{i}{D-\Omega}|\,0, \mathbf{z}\rangle R^{+}(0)\hat{O}^a_k R(0))\langle T/2, \mathbf{x}\,|\tfrac{i}{D-\Omega}|-T/2, \mathbf{y}\rangle\}\\
&\quad \times \langle T/2, \mathbf{x}\,|\tfrac{i}{D-\Omega}|-T/2, \mathbf{y}\rangle^2 D^{J}_{-T_3J_3}(R(-T/2))e^{\mathrm{tr}\log(D-\Omega)}. \qquad (5)
\end{aligned}$$

Here, the finite rotation matrix $D^{J}_{-T_3J_3}$, which carries the spin and isospin quantum numbers of the nucleon, appears due to the rotations $R(t)$ of the valence quark propagators in eq.(4) correlated by the $\Gamma^{\{g\}}_{JJ_3,TT_3}$ matrices and the integral over $\mathbf{z}$ is due to the translational zero modes treated in the leading order.

Now we are ready to make an expansion in Ω. For the effective action, it is well-known procedure [4,7,8,10], which yields up to the second order in Ω:

$$\mathrm{tr}\log\left(D - \Omega\right) \approx \mathrm{tr}\log D + i\frac{\Theta}{2}\int dt\Omega_a^2. \qquad (6)$$

Here Θ is the moment of inertia. The first term will be absorbed in Z whereas the second one gives the evolution operator acting in the space of matrix R. Expanding the quark propagator

$$\frac{1}{D-\Omega} \longrightarrow \frac{1}{D} + \frac{1}{D}\Omega\frac{1}{D} + ...$$

we can separate the zero order ($\sim N_c^0$) and the linear order ($\sim \frac{1}{N_c}$) corrections in Ω. The expansion in Ω is illustrated in Fig.1 a) and b) for the valence contribution and for the Dirac sea one, respectively.

Henceforward we will concentrate on the linear order terms. In this case we are left with the following path integral over R:

$$\int\limits_{R(-T/2)}^{R(T/2)} \mathcal{D}R D^{J}_{-T_3 J_3}{}^{*}(R(T/2)) \frac{1}{2}\,\mathrm{Sp}(R^{+}(0)\tau^{a}R(0)\tau^{b})\Omega_c(t) D^{J}_{-T_3 J_3}(R(-T/2)) e^{i\frac{\Theta}{2}\int dt \Omega_c^2}. \tag{7}$$

Here we use the identity $(R^{+}(0)\hat{O}^{a}_{k}R(0))_{fg} = \frac{1}{2}\,\mathrm{Sp}(R^{+}(0)\tau^{a}R(0)\tau^{b})(\hat{O}^{b}_{k})_{fg}$ in order to separate the $R(t)$ dependent part of the current which does not carry flavor indices fg. The path integral (7) can be taken [4] rigorously within the approximation (6). We obtain the well-known canonical quantization rule $\Omega_c \to J_c/\Theta$, where J_a is the spin operator. The final result for path integral (7) is a time ordered product:

$$\vartheta(-t)D_{ab}(R(0))J_c + \vartheta(t)J_c D_{ab}(R(0)). \tag{8}$$

which should be sandwiched between the nucleon rotational wave function. In order to obtain the result (8) we essentially made use of the basic feature of the path integral:

$$\int\limits_{q_1=q(T_1)}^{q_2=q(T_2)} \mathcal{D}q F_1(q(t_1))\cdots F_n(q(t_n))e^{iS} = \langle q_2, T_2 \,| T\{\hat{F}_1(q(t_1))\cdots \hat{F}_n(q(t_n)\}|\, q_1, T_1\rangle,$$

namely that the path integral can be equivalently written as the expectation value of the time ordered product of the corresponding operators. Using (8) and the standard spectral representation of the quark propagator it is straightforward to evaluate the matrix element of the current eq.(5). In particular for the linear correction we get

$$\langle N(p')\,|\Psi^{+}(0)\hat{O}^{a}_{k}\Psi(0)|\,N(p)\rangle^{\Omega^1} = \langle J, J_3T_3\,|[\frac{J_c}{\Theta}, D_{ab}]|\,J, J_3T_3\rangle$$

$$\times N_c \sum_{\substack{n>val \\ m\leq val}} \frac{1}{\epsilon_n - \epsilon_m} \langle m\,|\tau_a|\,n\rangle \int d^3z e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{z}} \Phi_n^+(\mathbf{z}) \hat{O}_k^b \Phi_m(\mathbf{z}). \tag{9}$$

Here Φ_n and ϵ_n are the eigenfunctions and eigenvalues of the single-particle hamiltonian h. Since the τ-matrix element is asymmetric with respect to exchange of the states m and n a commutator appears in the collective matrix element of eq.(9). It can be easily calculated:

$$\langle J=1/2, J_3T_3\,|[\frac{J_c}{\Theta}, D_{ab}]|\,J=1/2, J_3T_3\rangle = -\frac{1}{3}\frac{i}{\Theta}\varepsilon_{cb3}\delta_{a3}. \tag{10}$$

Using that both the axial coupling constant and the isovector magnetic moment are related to the corresponding form factors at $q^2=0$ finally we get for the $1/N_c$ rotational corrections:

$$g_A(\Omega^1) = \frac{N_c}{9}\frac{i}{2\Theta} \sum_{\substack{n>val \\ m\leq val}} \frac{1}{\epsilon_n - \epsilon_m} \langle m\,|\tau_a|\,n\rangle \langle n\,|[\boldsymbol{\sigma}\times\boldsymbol{\tau}]_a|\,m\rangle \tag{11}$$

and

$$\mu^{IV}(\Omega^1) = \frac{N_c}{9}\frac{i}{2\Theta} \sum_{\substack{n>val \\ m\leq val}} \frac{1}{\epsilon_n - \epsilon_m} \langle m\,|\tau_a|\,n\rangle \langle n\,|\gamma_5[[\boldsymbol{\sigma}\times\boldsymbol{x}]\times\boldsymbol{\tau}]_a|\,n\rangle \tag{12}$$

In both eqns.(11) and (12), a summation over a is assumed. As should be expected both expressions have similar structure with transitions from occupied to non-occupied levels and back. They include also an essential non-zero contribution from the Dirac sea. In contrast to the leading order the above expressions are finite and one does not need to regularize them.

The parameters of the model are fixed in the meson sector to reproduce $f_\pi = 93$ MeV and $m_\pi = 139.6$ MeV. Similar to other works [5,6,8] we use a numerical procedure based on the method of Kahana and Ripka [14]. The results for g_A and μ^{IV} up to the first order terms in Ω, are presented in Fig.2 and Fig.3 as a function of the constituent quark mass M. As can be seen in leading order (Ω^0) they are almost independent of

the constituent quark mass and the valence contribution is dominant. In the next to leading order (Ω^1) the valence and Dirac sea contributions show much stronger and quite different mass dependence: with increasing constituent mass M the valence part gets reduced whereas the Dirac sea part increases and becomes dominant. However, their sum shows almost no dependence on M. The result of Wakamatsu and Watabe for g_A (labeled as W&W in Fig.2) deviates from the present numbers. This is due to the fact that in their scheme the contribution of the Dirac sea to g_A in the linear order in Ω vanishes exactly. In the present calculations for both quantities the enhancement due to the $1/N_c$ rotational corrections improves considerably the agreement with experiment. In the case of g_A the experimental value is almost exactly reproduced and the inclusion of next order corrections will perhaps even overestimate it. In contrast to g_A for the isovector magnetic moment μ^{IV} we are still below the experimental value by 25 %. It is interesting to notice that the enhancement for both quantities due to the $1/N_c$ rotational corrections is very close to the estimate $\frac{N_c+2}{N_c}$ [15].

To conclude, we have evaluated the axial coupling constant g_A and the isovector magnetic moment μ^{IV} in the Nambu – Jona-Lasinio model in the next to leading order in the semiclassical quantization scheme. The $1/N_c$ rotational corrections are large enough to resolve the problem of strong underestimation of these two quantities in the leading order. In particular, g_A is almost exactly reproduced. However, such large linear order corrections imply that in order to control the perturbation series in Ω the next order ($1/N_c^2$) corrections should be investigated as well.

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FIGURES

FIG. 1. Diagrams corresponding to the expansion in Ω of the current matrix element: a) the valence contribution and b) the Dirac sea contribution.

FIG. 2. Axial vector coupling constant g_A evaluated up to the linear order in Ω as a function of the constituent quark mass M.

FIG. 3. Isovector nucleon magnetic moment μ^{IV} evaluated up to the linear order in Ω as a function of the constituent quark mass M.